ATTRIBUTES OF STUDENT MATHEMATICAL THINKING THAT IS WORTH BUILDING ON IN WHOLE-CLASS DISCUSSION

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This study investigated the attributes of 297 instances of student mathematical thinking during whole-class interactions that were identified as having the potential to foster learners' understanding of important mathematical ideas (MOSTs). Attributes included the form of the thinking (e.g., question vs. declarative statement), whether the thinking was based on earlier work or generated in-the-moment, the accuracy of the thinking, and the type of the thinking (e.g., sense making). Findings both illuminate the complexity of identifying student thinking worth building on during whole-class discussion and provide insight into important attributes of MOSTs that teachers can use to better recognize them. For example, 96% of MOSTs were of three types, making these three particularly salient types of student mathematical thinking for teachers to develop skills in recognizing.

Keywords: Classroom Discourse; Cognition; Instructional Activities and Practices

An enduring challenge in mathematics education is figuring out how to best support teachers' effective use of student mathematical thinking in their classrooms. For several decades reform documents (e.g., National Council of Teachers of Mathematics [NCTM], 1989, 2000, 2014) have consistently called for teaching that focuses on developing students' abilities to reason mathematically. For mathematical reasoning to happen, the NCTM recommends that students engage in exploration of complex tasks, state and test conjectures, and build arguments to justify their conjectures. In response to this recommendation, many researchers have investigated issues around student thinking, such as students' abilities to think mathematically using tasks with high cognitive demand (Stein, Grover, & Henningsen, 1996), obstacles to students' learning (Bishop, Lamb, Phillip, Whitacre, Schappelle, & Lewis, 2014), challenges beginning teachers face when trying to use student thinking (Peterson & Leatham, 2009), important teachable moments created by student thinking made public during classroom instruction (Stockero & Van Zoest, 2013), and classroom instances that have potential for building students' mathematical understanding (Leatham, Peterson, Stockero, & Van Zoest, 2015). However, little is known about the nature of student thinking that becomes publicly available for teachers to use during instruction.

Our ongoing work investigates student mathematical thinking made public during whole-class interactions that, if made the object of discussion, has the potential to foster learners' understanding of important mathematical ideas—instances of student thinking that we call **M**athematically Significant Pedagogical **O**pportunities to Build on **S**tudent **T**hinking [MOSTs] (Leatham et al., 2015). The work reported here analyzes instances of student thinking that have been identified as MOSTs in order to investigate attributes of this high-leverage subset of student thinking. A better understanding of the attributes of MOSTs has the potential to support research on mathematics teaching in at least four ways.

First, using student mathematical thinking productively requires that the thinking be noticed (van Es & Sherin, 2002). Stein, Engle, Smith, and Hughes (2008) suggested that teachers might be able to orchestrate classroom discussion effectively when student work with potential to enhance learning is

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identified, attended to, and sequenced in a developmentally appropriate way. Understanding attributes of MOSTs may help teachers develop their skills for noticing student thinking worth building on and thus improve their ability to orchestrate classroom discussion that fosters student learning. Second, Ball, Lewis, and Thames (2008) described students' mathematical thinking as "both underdeveloped and under development" (p. 15) and identified students' mathematical thinking as "the raw materials for building justified mathematical knowledge" (p. 25), but did not characterize the nature of the raw materials in student responses. Using students' mathematical thinking as a cornerstone for subsequent construction of student mathematical understanding requires an understanding of the nature of that thinking. Understanding attributes of MOSTs, a critical subset of student thinking, has the potential to provide insight into Ball et al.'s (2008) "raw materials" and contribute to the development of "justified mathematical knowledge" (p. 25). Third, Carpenter, Fennema, Peterson, Chiang, and Loef (1989) found that giving teachers access to different strategies students employ to solve problems positively affected teachers' beliefs about learning and instruction, their practices, their knowledge about students, and students' achievement. Giving teachers access to attributes of MOSTs may have similar positive effects on teachers because it would give them information about the nature of student mathematical thinking available to them in their classrooms and better equip them to use that thinking productively. Finally, Hiebert, Morris, Berk and Jansen (2007) argued that teaching should be assessed based on how teachers make use of student responses in classrooms to foster understanding of mathematical ideas rather than on the presence of recommended instructional features. Identifying attributes of student responses that are MOSTs might enhance the development of ways to assess teaching in this manner.

Theoretical Framework

Leatham et al. (2015) defined MOSTs as occurring in the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunities. For each characteristic, these authors provided two criteria that can be used to determine whether an instance of student thinking embodies that characteristic. For student mathematical thinking the criteria are: "(a) one can observe student action that provides sufficient evidence to make reasonable inferences about student mathematics and (b) one can articulate a mathematical idea that is closely related to the student mathematics of the instance—what we call a mathematical point" (pp. 93-94). The criteria for significant mathematics are: "(a) the mathematical point is appropriate for the mathematical development level of the students and (b) the mathematical point is central to mathematical goals for their learning" (p. 97). Finally, "an instance embodies the pedagogical opportunity characteristic when (a) the expression of a students' mathematics creates an opening to build on student thinking to help develop an understanding of the mathematically significant point of the instance and (b) the timing is right to take advantage of the opening" (p. 103). When an instance satisfies all six criteria, it embodies the three requisite characteristics and is a MOST. We see analysis of MOSTs as a means toward identifying important attributes of high leverage student mathematical thinking that might be used to help support teachers in developing their skill at productively using such thinking.

Stockero and Van Zoest (2013) investigated and categorized "instances in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding" (p. 127)—what they called pivotal teaching moments (PTMs). We see PTMs as a subset of MOSTs, and thus we used the PTM categories to inform our thinking about attributes of MOSTs. In particular, these researchers identified five categories of PTMs: (1) *extending*—students make connections to create a much deeper lesson from what was planned; (2) *incorrect mathematics*—student incorrect mathematical thinking becomes public; (3) *sense making*—students are trying to make sense of the mathematics under consideration; (4) *contradiction*—student responses have competing interpretations; and (5) *mathematical confusion*—students clearly state mathematically what they are confused about. These categories and the work related to the development of them provided a starting point for our exploration into attributes of MOSTs.

Methodology

This study is part of a larger project focused on understanding what it means for teachers to build on students' mathematical thinking (see LeveragingMOSTs.org). We selected 11 videotaped mathematics lessons from the MOST project that reflected teacher diversity (race/ethnicity, gender, experience, teaching style), mathematics diversity (6-12th grade, topic, textbook), and classroom diversity (region of the US, community type, race/ethnicity). The unit of analysis for identifying MOSTs was an instance of student thinking—an "observable student action or small collection of connected actions" (Leatham et al., 2015, p. 92) that had the potential to be mathematical. StudioCode (Sportstec, 1997-2015) was used for three passes of coding. In the first pass, classroom context and other relevant information were noted on the Studiocode timeline and instances of student thinking were identified and transcribed. During the second pass, the MOST Analytic Framework (see Leatham et al., 2015) was used to determine which instances were MOSTs. We identified 297 MOSTs in the 11 lessons; these MOSTs served as the data for the current study. The third pass of coding, completed for the current study, focused on identifying attributes of these MOSTs.

We coded the 297 MOSTs for seven attributes that fall into two groups: Locus and Cognition (Figure 1). The Locus group encompasses attributes that locate a MOST within the mathematical and lesson terrain and includes what immediately preceded the MOST (Prompt), whether the MOST was based on earlier work (Basis), and the distance of the mathematical idea of the MOST from the day's lesson (Math Goal). The Cognition group focuses on the expression of the student's thinking and includes whether the MOST was a question or statement (Form), whether the student thinking was correct (Accuracy), the extent to which the intellectual need is obvious (Intellectual Need), and the nature of the MOST (Type). To illustrate our coding and the nature of our results, we discuss four of the attributes (Basis, Form, Accuracy, and Type—bolded in Figure 1), as well as interactions between them.

Basis refers to whether the student mathematics (SM) in the MOST is based on earlier work (*Pre-thought*) or on in-the-moment thinking (*In-the-moment*). A MOST is coded *Pre-thought* when the student appears to be sharing thinking from previous work. Although this previous work could be from homework or another class, typically it is from small group or individual work completed during the lesson. A MOST is coded *In-the-moment* when the SM stems from students' in-the-moment thinking. This thinking might be in response to a follow-up request from the teacher or to another students' thinking or question, or it might be seemingly spontaneous.

Locus	Cognition
Prompt (Spontaneous, Open Invitation Spontaneous, Open Invitation Selected, Targeted Invitation) **Basis (In-the-Moment, Pre-Thought)** Math Goal (Lesson, Unit, Course, Math)	**Form (Question, Declarative, Tentative)** **Accuracy (Correct, Incorrect, Incomplete, Combination, N/A)** Intellectual Need (Obvious, Translucent, Hidden) **Type (Incorrect or Incomplete, Sense Making, Multiple Ideas or Solutions, Other)**

Figure 1: MOST Attribute Codes and their Categories by Groups

Form refers to the way in which the student thinking is expressed (*Question, Tentative Statement* or *Declarative Statement)*, regardless of its correctness or completeness. A MOST is coded *Question* if the student thinking is shared as a question or with the intent to question. *Declarative Statement* is used when students appear to be confident in what they are saying and *Tentative Statement* is used when the student appears to be making a conjecture or is wondering about something. Tentativeness is typically indicated by the student's voice inflection when making the statement, but it can also be indicated by their use of hedge words such as "maybe" or "I'm not sure."

Accuracy is used to categorize a MOST based on the validity of its SM. A MOST is *Correct* if its SM is a correct mathematical statement; *Incorrect* if its SM is an inaccurate statement; *Incomplete* if the SM is not incorrect, but it has gaps or ambiguities that keep it from being completely correct; *Combination* if it involves a complete statement(s) that falls in multiple *Accuracy* categories; or *N/A* if it is not possible to determine its correctness (e.g., if it is a question).

Type is used to categorize what about the SM made the instance a MOST. There are four *Type* categories: *Incorrect or Incomplete*, *Sense Making*, *Multiple Ideas or Solutions*, and *Other*. A MOST is coded *Incorrect or Incomplete* if it was compelling because its SM is inaccurate or missing critical components of the mathematical idea being expressed. A MOST is coded *Sense Making* if it was compelling because the SM implies that the student was trying to make sense of the mathematics, or they had comprehended an idea with which the class had been struggling. A MOST is coded *Multiple Ideas or Solutions* if it was compelling because the SM created an opportunity for comparison of multiple ideas or solutions.

To illustrate the attribute codes, consider the SMs from four MOSTs (see Figure 2). All four MOSTs came from class discussions based on tasks students had solved beforehand in small groups. SM1, SM2, and SM4 were in response to what was currently being shared rather than something they had done earlier, thus were coded *In-the-Moment (Basis)*. In contrast, SM3 was a reporting out of a student's earlier work, thus received the code *Pre-Thought (Basis)*. The first two MOSTs involved statements, rather than questions, thus received the code *Declarative Statement* (*Form)*. The third was also a statement, but the student preceded it by expressing a lack of confidence in her answer, thus it was coded *Tentative Statement.* The fourth was a question and was coded *Question*. In the *Accuracy* category, the first MOST received the code *Combination* because although it includes the correct idea that the rate of change is a constant $2.50, the language suggests that the slope is increasing at that rate rather than that the slope is that constant rate. The second MOST was coded *Incorrect* because it is possible to divide by a fraction. In SM3, also coded as *Incorrect*, the class had already agreed to the convention of putting the independent variable on the x-axis and the dependent variable on the y-axis and the problem the students were exploring implied that "money" was the dependent variable and "weeks" the independent variable. The fourth MOST was coded *N/A* for *Accuracy* because questions, by their very nature, do not have a truth-value. The compelling aspect of the first and fourth MOSTs was a student grappling with a mathematical idea—the difference

Student Mathematics of four MOSTs	**Coding (*Basis*, *Form*, *Accuracy*, *Type*)**
SM1: The slope is increasing at a constant rate. The slope is not going any faster. The slope is always going up $2.50.	*In-the-Moment, Declarative Statement, Combination, Sense Making*
SM2: You can't divide by a fraction.	*In-the-Moment, Declarative Statement, Incorrect, Incorrect or Incomplete*
SM3: I put the money on the x-axis and weeks on the y-axis.	*Pre-Thought, Tentative Statement, Incorrect, Incorrect or Incomplete*
SM4: Doesn't solving sometimes include simplifying?	*In-the-Moment, Question, N/A, Sense Making*

Figure 2: Coding Examples

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between an increasing graph and a graph with an increasing slope in SM1 and the difference between solving and simplifying in SM4—thus they were both coded *Sense Making (Type).* The compelling aspect of the second and third MOSTs was that the students had introduced incorrect ideas, thus these MOSTs were coded *Incorrect or Incomplete.*

Three research assistants individually coded the 297 MOSTs and then reconciled them as a group. If they were not able to reach agreement, the issue was brought to the attention of the principal investigators and either the codes or the code definitions were modified to resolve the issue. We then determined the frequencies of the codes and interactions between them for each of the 11 lessons and compiled all the results into a spreadsheet that allowed for within and across lesson comparisons. We used that information to search for patterns among the results that would lead to a better understanding of the attributes of MOSTs.

Results and Discussion

Figure 3 provides the percentages of MOSTs in each category of the four attributes *Basis*, *Form*, *Accuracy,* and *Type*. Roughly 20% of the 297 MOSTs in our data were based on work that students had completed earlier in the class, thus were available for the teachers to identify by monitoring students as they worked. This percentage speaks to the benefit of teachers developing skills such as the five practices for orchestrating classroom discussion identified by Smith and colleagues (e.g., Smith & Stein, 2011). The finding that 80% of the MOSTs were based on student thinking that occurred during whole-class interaction speaks to the importance of teachers also developing skills for carefully listening and responding to evolving thinking.

BASIS		FORM		ACCURACY		TYPE	
In-the-Moment	80	Question	16	Correct	40	Incorrect or Incomplete	31
Pre-Thought	20	Tentative	7	Incomplete	8	Sense making	50
		Declarative	77	Incorrect	24	Multiple Ideas or Solutions	15
				Combination	9	Others	4
				Not Applicable	19		

Figure 3: Percentages of MOSTs in Attribute Categories

The vast majority of the MOSTs were declarative statements (77%) as opposed to questions (16%) or tentative statements (7%). This means that it is insufficient to focus only on expressions of student mathematical thinking that are intuitively suggestive of thinking worth building on (such as questioning or wondering).Instead, teachers must develop more sophisticated ways of recognizing which student thinking has this potential.

Knowing the accuracy of an instance of student mathematical thinking is also insufficient to determine whether it is a MOST, as there was no predominate *Accuracy* category. There were, however, more correct (40%) than incorrect (24%)MOSTs. This is particularly interesting given that the project team had initially hypothesized that it would be difficult for correct student mathematical thinking to meet the MOST criteria.

Half of the MOSTs (50%) occurred when students were grappling to make sense of a mathematical idea. The next highest *Type* category involved instances of student thinking that were incorrect or incomplete (31%). Student thinking that led to multiple ideas or solutions being available for students to consider occurred in 15% of the MOSTs and 4% did not fit in the three main categories of *Type*. Although *Incorrect or Incomplete* and *Multiple Ideas or Solutions* had lower frequencies than *Sense Making*, they may be easier for teachers to recognize. Thus, it seems important for teachers to attend to all three main categories of *Type*. The fact that 96% of the MOSTs were captured by these three categories is encouraging as it suggests some parameters for developing teachers' abilities to recognize MOSTs.

Figure 4 considers interactions between the attributes. All but one of the MOSTs in the form of questions (see the Q column in Figure 4) were compelling because the student was grappling to make sense of a mathematical idea (see, for example, SM4 in Figure2). The fact that all questions in this data that qualified as MOSTs involved sense making suggests the need for teachers to consider the potential of a question to determine the nature of their response. If the question is a MOST, the most effective teacher response might be to provide an opportunity for the class to join the student who asked the question in making sense of the idea.

	FORM			BASIS		ACCURACY*				
	Q	T	D	ITM	PT	CR	ICR	INC	COM	N/A
TYPE										
Incorrect or Incomplete	1	12	87	80	20	0	66	22	11	1
Sense making	32	5	63	83	17	58	1	3	3	35
Multiple Ideas or Solutions	0	2	98	71	29	58	16	0	24	2
Others	0	18	82	91	9	55	9	0	9	27
FORM										
Question (Q)				98	2	0	0	0	0	100
Tentative (T)				91	9	27	45	5	23	0
Declarative (D)				76	24	49	27	10	10	4
BASIS										
In-the-Moment (ITM)	20	8	72			36	25	9	7	23
Pre-Thought (PRE)	2	4	94			55	19	5	17	4

*Accuracy Abbreviations: Correct (CR); Incorrect (ICR); Incomplete (INC); Combination (COM); Not Applicable (N/A)

Figure 4: Percentages of MOSTs in Interactions between Attribute Categories

The MOSTs that were compelling because they provided an opportunity for students to consider multiple ideas or solutions were predominantly declarative statements (98%). As might be expected, MOSTs in which students shared thinking from previous work were also typically declarative statements (94%). Still, 76% of the declarative statements resulted from in-the-moment thinking, as did 98% of the questions and 91% of the tentative statements. There were also no extreme differences among which *Types* result from in-the-moment thinking, with *Sense Making* having the highest frequency (83%) of in-the-moment thinking of the three main *Type* categories and *Multiple Ideas or Solutions* having the lowest (71%). Thus the earlier stated need for teachers to skillfully respond to evolving thinking stands, regardless of the form in which that thinking is expressed or what made it compelling.

Figure 4 also shows a much higher percentage of the tentatively stated MOSTs were incorrect as were correct (45% vs. 27%); nearly the reverse was true of the MOSTs that were declarative statements (27% vs. 49%). This finding suggests that although there was some correlation between students' confidence in their thinking and the accuracy of it, the relationship was not strong enough to be counted on. That is, in the context of MOSTs, relying on tentative thinking to be incorrect and confident thinking to be correct would cause one to be wrong much of the time. Likewise, although pre-thought SM was more likely to be correct than SM that was generated in the moment (55% to 36%), both types of SM were also often incorrect (19% and 25%, respectively).

Finally, with the exception of MOSTs that were compelling because they involved incorrect or incomplete thinking (e.g., SM2 and SM3 of Figure 2), there did not seem to be a strong relationship between *Accuracy* and *Type*. For example, although 58% of MOSTs that provided the opportunity for students to consider multiple ideas or solutions were based on correct SM, 24% were based on SM that had both correct and incorrect elements, and 16% were based on SM that was incorrect. Again, it seems that accuracy of student mathematical thinking is not a useful predictor of MOSTs.

Conclusion

This study set out to contribute to our developing understanding of how to best support teachers' effective use of student mathematical thinking in their classrooms by investigating attributes of MOSTs—a high leverage subset of student thinking. The results provide insight into claims about the complexity of responding to students' mathematical thinking on the spot (e.g., Choppin, 2007; Jacobs, Lamb, & Philipp, 2010). We now know that surface features of thinking, such as how it occurs, the form in which it is expressed, and how accurate it is, are not sufficient to determine whether the thinking should be pursued. Rather, responding effectively to student mathematical thinking requires careful attention to the content of the thinking to discern the underlying mathematical idea and what it might offer as the object of a class discussion. For example, some student questions may be best answered directly, but those that reflect a student's grappling with important mathematical ideas provide rich opportunities to engage the class in the type of mathematical activity advocated by current reforms (e.g. NCTM, 2014). Calculation and other surface mistakes may be dispensed with quickly, but errors in students' thinking are often worth building on. Similarly, correct answers may be an indication to continue, or they may provide an opportunity to stop and engage the class in consolidating important mathematical understandings.

Despite the lack of easy answers about which thinking is worth building on in whole-class discussion, this work does provide some parameters that may make the process more manageable. For example, correct student thinking that does not involve sense making or multiple ideas or solutions is not likely to be worth pursing in a whole-class discussion. Being aware of this pattern can help teachers avoid initiating unproductive discussions.

In general, this work supports the need for teachers to have criteria they can use for evaluating which student thinking is worth building on. The MOST Analytical Framework (Leatham, et al., 2015) is one such set of criteria. Such criteria, in conjunction with the parameters contributed by this study, provide a starting place for designing teacher education and professional development to support teachers in developing the teaching practice of productive use of student thinking.

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